

Application to
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We appologize before hand if some links are unavailable. We will forward the correct link upon request. A (partially) French version of this text can be available on demand and will be posted on the network site <http://www.univ-irem.fr/spip.php?rubrique379>

1 Nominee's programme and reasons for the nomination

The IREM network (Research institute for mathematic education) was created in France 50 years ago. Since then, the IREM have contrived to be one of the most active and unavoidable actors in mathematics education in the country, by responding to the three missions entrusted to them:

- research to improve the teaching of mathematics in non-hierarchical groups of university researchers and teachers of the first or second degree school or in higher education;
- Train teachers, especially using the results of network research
- Disseminate the results of research in mathematics, mathematics education or history.

These missions, and the originality of their university structure, associating all the actors of the world of education in mathematics, are at the origin of many productions. Those works are always close to the concerns of the field professors, adapted to the teaching issues arising from the many curricular evolutions and to the emergence of new technological tools.

The IREM, each with their local specificity, but with constructive scientific coordination, form a network that is both adaptable and coherent. They are recognized in France and in many countries, particularly French-speaking countries, or having natural research relations with France. Some of those have in turn created or are seeking to create structures of IREM type.

The IREM are plebiscited both by researchers in mathematics and didactics and by field teachers; In particular, they latter use in confidence the resources developed in the network and disseminated by an evolving system of common or local sowing media which will be detailed in this folder. It is now used as a model for improving the teaching of other sciences, in particular STEM (Science, Technology, Engineering and Mathematics) , which is being increasingly welcomed in the network. This is an explicit demand from both the Ministry of National Education, the scientific universities, and the researchers on other sciences education fields.

It seems to us that the fiftieth anniversary of the IREM network is the opportunity to apply to the prestigious Emma Castelnuovo award, and highlight the originality, the effectiveness, and the implication of its structure, the quality and the diversity of the work done, the tight links weaved between practice and theory. We will show influence of the network on teaching practices and its potential to be used as a model in other countries and for other sciences.

1.1 The fundamental principles of work in the IREM: a close network at the teachers service

The second half of the twentieth century has been marked, in France as elsewhere, by the need to renew strongly the teaching of mathematics, to adapt to the considerable evolutions of the knowledge and the conceptions carried out by the researches in this field. The IREM are thus born of the conjunction of a great disarray of the teachers, following the so-called "modern mathematics" curricular reform, and the events of May 68, shaking among others the academic world. This creation, by the french government, was solicited both by the Association of Mathematical Teachers of Public Education (APMEP), which was already developing training and resources to anticipate and support the "retraining" in mathematics teachers, and by faculty-researchers aware of their responsibilities.

Many mathematics teachers then brought in their enthousiasm for this militant work, close to their professional concern. The IREM then furnished them, inside the universities, with a new working environment, stimulating, within groups, away from hierarchical overview.

The three primary missions of the IREM (see above), underpinned by the conditions of their creation, have thus led to fundamental constants, in time and space, in their functioning:

- **A research activity** inside universities, throughout France, within working groups of "research-action". These group associate researchers and teacher-researchers in mathematics, mathematics

education or mathematics history, as well as field teachers, each of them bringing in his own knowledge and expertise, sensitivity and questioning. In 2018, 263 groups, spread in the 27 IREM, gathered 1840 members, among which 463 faculty members. The diversity as well as the great number of groups then allow the variety of the research themes and types of production.

This diversity of status and the reciprocal trust allow for productive classroom observation and critical thinking, and important professional development for all members. These research works are supported by universities, by local authorities (rectorates, inspectorates) and national authorities (Ministries of Education and Higher Education and Research), by the provision of work spaces, financial and, above all, human resources.

- **Training for all teachers** "from kindergarten to university", proposed by working group members, ensuring subtle consideration and appropriation of work proposals. The contents are from research of the intervening group and/or other IREM groups with whom they may have interacted, or from theoretical research, suggested by the researchers of the group.

In 2018, 181 training sessions were held, for 5 010 participants, corresponding to 21 151 half-days of trainee training, to which one has to add 7 328 of training to competitive exams for internal promotions. Pre-service training of professors is also impacted thanks to the presence in the groups of trainers from university institutes in charge of these training courses, who often began as trainers for their IREM.

- A multiplicity of modes of **transmission of research results**: direct training; drafting of research articles, publications in interface journals, publication of booklets, books, various resources, in paper and/or online; provision of books and resources in IREM or academic libraries, on IREM sites or on the network site (<http://www.univ-irem.fr/>);

The network publishes 4 peer-reviewed journals:

- *Repères IREM*, a generalist journal that publishes the work of the network and special thematic issues <http://www.univ-irem.fr/spip.php?rubrique23> ;
- *Grand N*, for primary school teachers and their trainers (edited by the IREM of grenoble, <http://www-irem.ujf-grenoble.fr/spip/spip.php?rubrique13>);
- *petit x*, for secondary school teachers and their trainers (edited by the IREM of grenoble, <http://www-irem.ujf-grenoble.fr/spip/spip.php?rubrique12>);
- *Annales de Didactique et de Sciences Cognitives* (Annals of Didactics and Cognitive Sciences), research review, (<https://mathinfo.unistra.fr/irem/publications/adsc/>, edited by the IREM of Strasbourg).

The resources on mathematics education are reviewed on the "*Publimath*" server (co-managed with the APMEP, <http://publimath.univ-irem.fr/>) by creating reading sheets and linking them to available digital versions of the resources. For the 50th anniversary of the network, two old or recent resources are highlighted each week with a wide electronic distribution (twitter and <http://www.univ-irem.fr/spip.php?rubrique513>).

- The organization of **conferences**, either local, national (4 to 6 per year gathering 100 to 250 people each), or even international conferences on mathematics education or their history (for example the organization of HPM 2016, by the IREM of Montpellier (<https://hpm2016.sciencesconf.org/>)); the organization or participation in numerous events to disseminate and promote mathematics to scholars and to the general audience: mathematics week (annual, national), fortnight of the science festival (annual), mathematics year (2013 and 2020), mathematics game fairs, class hosting, internships and extracurricular actions...

1.2 A strong synergy between IREM

The synergy between the different IREM, each with its own status, type of local integration, but no centralized management of the structures, is ensured by three types of assemblies, bringing together IREM members and representatives of other bodies.

- **The IREM Directors Assembly (ADIREM)**

Four times a year, it gathers the directors, the chairman of the scientific committee, and representatives of friendly associations: Commission Française pour l'enseignement des mathématiques (CFEM, french section of ICMI), APMEP, Société Française de mathématiques (SMF), etc. It coordinates administrative and political aspects of the network and represents it in front of national authorities.

- **The Scientific Committee of the IREM** (<http://www.univ-irem.fr/spip.php?rubrique71>)

It is currently made up of 20 people, half representing the IREM and half external observers (including non-mathematicians). Its role is to observe the network's activity, evaluate its work, generate perspectives and contribute to the IREM's voice. At each of its meetings, it organizes debates on current topics concerning mathematics education.

- **The Inter-IREM Commissions (CII)** (<http://www.univ-irem.fr/spip.php?rubrique3>). They bring together members from different IREM to work on given themes, compare the work of IREM, encourage specific research, produce common resources, organize conferences and meetings, and reflect and anticipate curricular developments. They are a source of proposals thanks to their expertise for national or international actions.

There are currently 13 CII of several types:

- 4 of them are associated with a sector of the educational system: CII Collège (junior high-school, grade 6 to 9), CII Lycée (highschool, grade 10 to 12), CII Lycée Professionnel (vocational high school), CII University;
- 2 are dedicated to pre-service (and in-service) training: COMmission de Recherche sur la Formation des Enseignants de Mathématiques (CORFEM) for training of secondary school teachers, Commission Permanente des IREM sur l'Enseignement Élémentaire (COPIRELEM) for training of primary school teachers; They each organize an annual congress for teachers trainers;
- 3 of them are working on transversal themes: CII Epistemology and History of Mathematics (which organizes a congress every 2 years), CII Didactics of mathematics, and CII Information and communication Technologies for teaching (TICE);
- One is specialized in informatics (CIII) (currently partially taught by mathematics teachers);
- Two are dedicated to the network resources sharing: CII Publimath, share with APMEP, which manages the *Publimath* server, and CII *Repères-IREM*, which is the editorial committee of the journal;
- And the very new International CII (see 3.7)

The number of CII evolves according to needs. The commission for the popularization of mathematics (CII Pop'Maths) stopped in March 2019 following the publication of its book *panoramath*'7 (cf 3.6). Other commissions will soon have to be created, following the increasing opening of the IREM to groups and activities concerning other sciences (sometimes without any mathematician).

In addition, the network (ADIREM or CII) organizes 4 to 6 national or international symposia each year, in which network members and external persons participate. These congresses are locally organized by one of the IREM of the network. For instance, in 2019, ADIREM organizes the 50th anniversary symposium in Besançon (<http://www.univ-irem.fr/spip.php?rubrique512>), the CII Epistemology and History organizes a congress in Poitiers (<http://www.univ-irem.fr/spip.php?rubrique158>, registered in the national training plan for teachers (PNF)), and the annual congresses of the CORFEM and COPIRELEM, respectively in Strasbourg and for the first time in Switzerland at Lausanne (<http://www.univ-irem.fr/spip.php?rubrique120> and <http://www.univ-irem.fr/spip.php?rubrique442>). In 2020, there will also be a symposium on "Mathematics and Language Education" in Clermont-Ferrand and a symposium of the CII TICE in Marseilles. Proceedings of the conferences are published and put online on the IREM network site, accompanied by some videorecordings of interventions.

1.3 Space of mathematics education integration

The ADIREM is associated with APMEP (Association of Mathematics Teachers of the Public Education Service), the scientific societies of mathematics (SMF, SMAI, SFdS, ARDM), the Academy of Sciences, the General Inspection of Mathematics, and national associations for the promotion or dissemination of mathematics: Women & Maths (<http://www.femmes-et-maths.fr/>) and MATH.en.JEANS (<http://www.mathenjeans.fr/>). All together, they form the French section of ICMI: the French Commission for Mathematics Education (CFEM, <http://www.cfem.asso.fr/>).

The APMEP, which contributed to the creation of the IREM, continues to edit or co-edit some of the resources from the groups. Many exchanges take place during IREM symposia and "APMEP Days" (training congress organized by the APMEP). The IREM, thanks to their university structure and local connections, organize congresses for MATH.en.JEANS or specific actions for Women & Maths (see 3.6). They organize an annual challenge <http://irem.u-bourgogne.fr/challenge-graines-de-sondeur.html> with the French Statistic Society (SFdS).

The members of the Association for Research in Mathematics Didactic (ARDM, <https://ardm.eu/>) are mostly also members of IREM groups that benefit from their theoretical knowledge and provide them with a strong anchorage within the field (cf 3.3).

The network has been several times associated with the French Academy of Sciences for works or conferences on teacher training. In 2018, a symposium on teaching at primary school (<http://www.fondation-lamap.org/fr/page/65880/colloque-sur-lenseignement-des-mathematiques-a-lecole-primaire>) was co-organized with the foundation for sciences education *La Main à la pâte* (www.fondation-lamap.org), emanation of the Academy of Sciences, which relies locally or nationally on the IREM's expertise for its activities with schoolchildren and teachers training in mathematics, and in return disseminates its resources. The network participated in the work and fed into the reports of the CREM (<http://educmath.ens-lyon.fr/Educmath/ressources/etudes/crem>, Commission for Reflection on Mathematics teaching), which was chaired by the academician Jean-Pierre Kahane, President of the IREM Scientific Committee from 1999 to 2003. This commission was created in 1999 by the Ministry of Education to work upstream of mathematics programmes.

The Inspectorates of Mathematics Teachers (national and regional) who contribute to the development of curricula and teacher training, have been very abounded by members of the IREM groups, thus participating in the irrigation of the mathematics teachers by the works produced in network. Resources have been co-produced by the General Inspection and the IREM and thus have the status of official resources (see <http://eduscol.education.fr/cid99696/ressources-maths-cycle.html>). Xavier Buff, then Director of IRES Toulouse (Research Institute Sciences Education, including mathematics), participated in the work of the Superior Council of Programmes, which defined the 2015-2016 programmes currently in force in primary schools and junior high school. The IREM are currently being consulted with the CFEM before the implementation of the new high school and vocational high school programmes.

The IREM also were interviewed by the Villani-Torossian Commission for its official report (February 2018, <https://www.education.gouv.fr/cid126423/21-mesures-pour-l-enseignement-des-mathematiques>) on the teaching of mathematics in France, following the bad results of France in international surveys. This report has repeatedly pointed to the IREM's preponderant role in teacher training. Christian Mercat, Director of the IREM of Lyon, was a member of the commission. Several proposed measures are explicitly based on the IREM, and in particular the two measures currently in place: circonscription referents in mathematics and the mathematics laboratories in the high schools, which working conditions reproduce the IREM groups. The IREM are particularly active in supporting the implementation of these two measures, and a number of these new missions are being entrusted to members of IREM groups.

Finally, ADIREM is part of the steering committee for the Year of Mathematics 2020, set up by an agreement between the National Scientific Research Center (CNRS), co-tutelle for the research laboratories of the universities that host the IREM, and the Ministry of National Education.

1.4 An open in permanent evolution and active on educational developments network

Being university institutes, the IREM are flexible autonomous structures. They have kept up the militant momentum of their creation and continue to anticipate, criticize and/or support perpetual developments, in particular curricular reforms (3 reforms in 10 years), affecting the teaching of mathematics in France. The Scientific Committee, through its debates, the international network, by the diversity of its experiences, the CIIs and the research groups through their work, are the strength of the IREM in the face of these developments.

In recent years, these developments have focused in particular on: the growing importance of modelling and interdisciplinarity; the introduction and subsequent removal of many notions of probability and statistics; the repeated changes in the teaching of geometry; the increasing and diversified use of digital technologies; the modification of the methods of evaluation; the vocational education specific needs improvement, whose place has increased; the introduction of algorithms at all levels, and then of computer science, and now teacher positions in computer science; changes in teachers' working habits (setting up cycle councils between primary and secondary schools, mathematics laboratories in secondary schools) and in their in-service training (setting up "referents in mathematics" to support in-service training in small groups for primary school teachers).

The current developments (in particular the last two quoted) result from the Villani-Torossian report (see 1.3). They are relating to in-service teacher training and attempt to generalize IREM groups in a more local framework. They are largely supported locally and nationally by the IREM, as key players capable of making the link with all the professionals affected by these changes, and having expertise on their adaptability. Anne Cortella and François Recher, directors of the IREM of Montpellier and Lille, for example, participated in the writing of the *vademecum* defining those new training modalities (<http://www.univ-irem.fr/spip.php?article1511>).

The IREM also act together with the APMEP and of the learned societies (SMF, SMAI, SFdS) within the CFEM, the undisputed interlocutor of the Ministry of National Education, in the implementation of new programmes and working organization in high school: for example against the near absence of mathematics in the new common core of education, and to make it possible to choose a different mathematical content for students who are not destined for scientific studies.

At last, the network is very concerned about the aggravation of the unequal character of French education, particularly in mathematics, as evidenced by international evaluations. It is also concerned about the growing defection from scientific studies and the lack of students for mathematics teaching careers. In addition to its core missions, the network develops actions to disseminate and promote mathematics and scientific careers for the youngest, and in particular those with a low social level.

2 Summary statement

Created 50 years ago, the IREM (Instituts de Recherche sur l'Enseignement des Mathématiques) network rapidly became an essential and highly recognized actor for mathematics education in France, for all school levels. Today, this network includes 27 IREM in France and also similar structures in at least 9 other countries. Although similar structures were never created for other disciplines in the country, the IREM include more and more interdisciplinary and STEM disciplines.

The IREM are university structures, close to mathematic research departments but autonomous, with three main missions: research, especially action-research addressing questions emerging from practice and curricular evolutions, professional development activities based on research, and production and dissemination of mathematical knowledge and resources for teaching and for teacher education.

Their functioning is based on original principles, which were set from the beginning, and have made possible their achievement. The most important one is the pioneering principle of collaborative work of different status teachers and mathematicians, freed from any hierarchical relationship, within thematic research groups. These are created to address a particular problem and gather a diversity of expertise over several years: primary and/or secondary mathematics teachers, university mathematicians, historians and didacticians of mathematics, teachers educators.

In 2018, the 27 IREM counted 263 such groups and 1830 members (1/4 working in universities, and 65% being secondary teachers). No one works full time in an IREM, and IREM's achievements owe much, and more and more, to IREM members commitment and volunteer work. These constitute an open and moving community as new groups are regularly created while others end their projects. Participating in IREM activities stays the essential mode for mathematics teachers professional development, and also for teachers educators professionalization. Many of them, as for inspectors of the education system, have benefitted from this system.

The IREM have played a major role in the emergence of what is known as the French school of didactics of mathematics, and they constantly accompanied and influenced its development. They have also nurtured the emergence and development in France of a strong community of research on history and epistemology in mathematics education, very active in the ICMI affiliated group HPM.

The network is administrated by the Assembly of IREM Directors, also in charge of relationships with partner institutions, especially with the Ministries of Education and Research. The Scientific Committee of the network has reflexive and prospective aims. The currently 13 inter-IREM Commissions are gathering representatives of different IREM on specific themes to organize the collective work, national events and publications. The network international contacts are extending, leading to the creation of a new International Commission. This organization allows to cumulate the IREM's respective achievements, and to globally benefit from their respective work and means. The large-scale dissemination of IREM activities and outcomes is also supported by the publications of the network, among which four widely distributed journals acknowledged by national agencies. In 2018, the IREM network organized about 200 local professional development activities attended by 5000 participants, regular seminars, 4 national congresses, and contributed to many events organized by partner institutions and the Ministry of Education. It published 7 issues of journals, 10 books or conference proceedings, and more than 95 resources for teaching and teacher education either in paper format or on-line, and created 4000 reading notes. Most of the productions are now digitalized and available on-line.

For 50 years, the IREM have had to constantly defend their original structure and vision of mathematics education, as both a field of research and practice, their collaborative vision of professional development. Thanks to the mobilization of a number of actors, to the network solidarity, they always have overcome obstacles, and survived. They are today a dynamic and mature structure, constantly questioning itself but seeing in the international evolution an encouragement to pursue with their vision.

3 Account of the genesis and dissemination of the nominee's work and role of the people involved

The IREM were born out of the strong need for in-service training for mathematics teachers due in the 1960's by the massification of secondary education and the "modern mathematics" reform (influenced by Bourbaki). Until then foreign to the teachers' culture, the APMEP (1.3) had begun work on the development and dissemination of resources on these "modern" branches of mathematics and requested these creations. Their founding principles (see 1.1) have made it possible to institutionalize, massify and perpetuate these actions. The network continues, in more and more fields, to develop resources on mathematics (or science) education, and organize their dissemination, offering teachers many opportunities for training and professional development.

The IREM's influence takes many forms : research in working groups and CII with local dissemination of their work; publications developed in the IREM or in the network; strong links with research in didactics, epistemology and history of mathematics; impact on initial teacher training ; the network's involvement in curriculum development and concern ; actions to popularize mathematics; international contacts ; openness to other sciences. All these actions involve a large number of people, academics, professors, teacher trainers, inspectors. At the end of this section we give their roles and introduce some of the "key people" among many others.

3.1 Working groups: main place for the elaboration of the resources and their local dissemination

Researches on mathematics education in the IREM are carried out in working groups, whose members (called "facilitators") are teachers (primary, secondary) and researchers in mathematics, didactics, history or epistemology of mathematics, but also teachers and researchers in other disciplines (computer science, physical science, technology, biology, economics, philosophy). These groups can be considered as places of training and professional development for all their members. The research gives rise to numerous local or national publications (see 3.2).

In 2018, the 1 840 facilitators of the 27 IREM gathered regularly, with very small compensation: primary school, junior high school and high schools teachers (about 65% à 70%) and university faculties (about 20%). The 13 Inter-IREM Commissions (CII, cf. 1.2) have gathered 219 of them in order to organize, distribute, exchange on the groups work and produce synthesis or original resources.

The in-service training courses offered to teachers in the IREM are run by the members of the groups, which ensures their scientific and pedagogical quality. They can rely on the many publications resulting from the work of their group or other groups, their IREM or an other IREM, as well as on the synthesis work resulting from CIIs (booklets, books, conference proceedings), and on the work of didacticians or epistemologists from the network (see 3.3). In 2018, 330 training days received 5 000 trainees, for a total of 14 000 half-days of trainee training (hDTT), i.e 7% of secondary school teachers, fewer elementary school teachers. Some IREM also support preparation for body change competitive exams (12 internship in 2018 for 300 trainees and 7 500 hDTT). In addition, training is provided at national and local congresses, seminars, or conference cycles, by an IREM, the ADIREM (Assembly of IREM Directors, see 1.2) or a CII. In 2018, six national congresses for 13 days brought together 644 delegates (3 600 hDTT) and more than fifty conferences and seminars were also organized.

It should be noted that the demand from teachers, who highly appreciate the training provided by the IREM, as well as the supervision they offer, is for a substantial increase in internship training days. But the resources granted by the administration for continuing training are constantly decreasing, and IREM internships are often cancelled when curriculum reforms occur. Also are only granted short-term internships (1 or 2 days, or online) when a longer time should be required to allow a better

impact. Nevertheless, online training courses (mooc, m@gistère) that summarize the internships are offered in conjunction with the IREM to allow more time: for example, the mooc e-fan at the IREM in Lyon with the ENS (<https://www.fun-mooc.fr/courses/course-v1:ENSDeLyon+14003+session05/about>). The IREM network is campaigning, with the help of other societies, in particular the APMEP and the Academy of Sciences, for whom it is a priority objective, to significantly increase the possibility of in-service training for teachers.

The Villani-Torossian Commission's report (see 1.3) has also pointed this lack of in-service training in France, and proposes several measures, some of which are inspired on both IREM groups and lesson-studies, and rely on the IREM network (see measures 15-16-27-28).

3.2 The publishing policy of the IREM network

Since their creation in 1969, the IREM have had an active publishing policy. It was unusual then, for teachers, to be encouraged to participate in writing activities related to their teaching. One of the original features of the IREM was to involve them in working groups, accompanied by university faculties, familiar with the scientific writing, with the aim of publishing booklets or articles resulting from these activities, for the other teachers. The written works mass thus collected in the IREM is considerable. They had a formative role when they were published and are a documentary source of great interest.

IREM publications are aimed at mathematics teachers (first and second degree, general or vocational education, higher education), teacher trainers, researchers in didactics, history or epistemology of mathematics. They are used by students in their initial professional training or for their initiation to research. They are of interest to anyone concerned with pedagogy or educational research. The themes addressed are diverse and reflect the wide variety of IREM missions on mathematics education from kindergarten science to university: reports on classroom activities, history of mathematics, popularization of mathematics, connections with other disciplines...

Here is a classification of IREM productions, with information on their number for 2018

- "Booklets" (20 in 2018) from 50 to 100 pages; used to publish the work of a group in an IREM, as those of Poitiers, one of which is presented in 4.3 (see http://irem2.univ-poitiers.fr/portail/index.php?option=com_content&view=article&id=180&Itemid=197);
- Books (11 in 2018): used to publish the work of an CII around a work theme that it has set for itself or to publish the proceedings of symposia;
- Articles (48 in 2018), some published in the network reviews (see below), or in the reviews of CFEM partners (see 1.3). are used to synthesize the production of a working group or some of its members on all or part of their work.
- Hundreds of "Working Documents" per year, often in digital form and available on-line on an IREM website; their contents reflect the work evolution within a group; they are a training to writing and serve as a pre-publication.

These works are referenced by the CII *Publimath* and its search engine, common to IREM and APMEP, that establishes for each production a form that provides information on editorial requirements and content of the document. It provides a link to a downloadable version of the document if possible (<http://publimath.univ-irem.fr/>). *Publimath* references also publications reported as being useful for the teaching of mathematics in the French-speaking world. Among the 29 000 reading sheets filled by *Publimath*, 8 250 come from an IREM, including about 3 000 booklets or books: about 600 for elementary education, 1 000 for junior high school, 1 000 for high school. They also include about 100 videos, chapters in conference proceedings, journals articles and various texts available on the websites of different IREM. On average, since its creation, each IREM has published 90 booklets or books, 150 journal articles and 300 other resources (classroom situations, online courses, videos).

Some previous resources have been revisited recently: either to rework them with the Mathematics General Inspectorate (<http://eduscol.education.fr/cid99696/ressources-maths-cycle.html>) to make them official resources; or to honour them for the IREM's 50th anniversary by publishing weekly short links between an old and a new production, or with regard to current events in mathematics education (<http://www.univ-irem.fr/spip.php?rubrique513>). In addition, there are software and video resources (for example those of the IREM in Paris (<https://irem.univ-paris-diderot.fr/videos-de-lirem-de-paris>)). At last several theses are defended each year by members of the network, fed by the works of the groups.

The network publications have been digitized and are available either in paper or in digital form for free online access on the IREM or CIIwebsite or via *Publimath*. The "booklets" and books are edited by an IREM itself, as part of a collection within a university press department (those of Franche-Comté (PUFC) or Limoges (PULIM)) or by private publishers (CII books for wider distribution).

The 4 journals of the IREM network below have national recognition by the High Council for Evaluation of Research and Higher Education (HCERES, <http://www.aeres-evaluation.fr/>) which has classified the first 3 in the "Interfaces" category, and the last in the "Research journals" category. They are freely accessible from their websites within two to three years following publication. They are pre-service and in-service teachers training implements, as well as trainers and researchers implements.

- *Repères IREM* (<http://www.univ-irem.fr/spip.php?rubrique24>), created in 1990, 114 issues = 4 per year, 1007 articles published. Its purpose is to serve as an interface between the research community, at a national level or in French-speaking countries. It informs the actors of the mathematical educational environment, but also of the related disciplines, about the work and reflection carried out jointly between teachers and researchers, in the classroom or in teacher training (of the first, second or higher degree). It focuses on current issues that affect the teaching communities: investigative approaches, interdisciplinarity, taking into account the pedagogical aspects of disability, assessment by competence, etc..., whether they relate to major debates or more simply to practical applications.
- *Petit x* (<http://www-IREM.ujf-grenoble.fr/spip/spip.php?rubrique25>), created in 1983, 107 issues $\simeq$ 3 per year, 10 articles per issue, 1082 articles; published by the IREM of Grenoble, sponsored by ARDM (cf 1.3) and ADIREM. It is a review of mathematics didactics and of teaching practices analysis, at the secondary school level or at transitions (primary/junior, junior/high school, secondary/post-baccalaureate). It disseminates research, reflections, analyses and reports of work and class activities. The published articles contribute to ensure that this research and teaching practices feed into each other. *Petit x* also pursues international exchanges in the French-speaking world in the field of mathematics education, with the help of the foreign members of its editorial committee. It encourages young French-speaking researchers to present articles, of which a significant number are published.
- *Grand N* (<http://www-IREM.ujf-grenoble.fr/spip/spip.php?rubrique13>), created in 1973, 103 issues $\simeq$ 2 par an, ± 12 articles per issue (1884 articles) and special sections; published by the IREM of Grenoble, supported by the COPIRELEM (cf 1.2). Initially dedicated to mathematics teaching in primary school, it has been enriched since 1990 by the contribution of other scientific disciplines, and is the only French journal specifically dedicated to the sciences in education. Its reading committee represents all the training bodies of primary school teachers.
This review is at the interface between the field of research and that of professional practices. Its contents, anchored into research on mathematics and science teaching and in teacher training, aim to make it a real implement at the service of teachers and trainers, by broadening their field of possibilities in terms of classroom situations and by studies on the transition from school to college, as well as by situations that can be transferred to college.
- *Annales de Didactique et de Sciences Cognitives*, created in 1988, 23 issues, 6 articles per issue (<http://mathinfo.unistra.fr/IREM/publications/ADSC/#c62294>). It is an annual review of the

IREM in Strasbourg. It publishes research results suitable to develop and stimulate reflection on mathematics education for all types of audiences (schoolchildren, high school students, students and adults in training): research concerning the pre-service and in-service training of teachers; on teaching in various socio-cultural contexts; combining a didactic theoretical framework and experimentation in the context of teaching; syntheses of research conducted in a particular field. The theoretical reference domains come from mathematics education but can also be based on cognitive psychology and linguistics.

Some IREM also publish or published a local journal, like *Vine Leaves* (IREM of Dijon, 130 numbers, until 2014) or *The Open* (IREM of Strasbourg, until 2010, 118 numbers, 606 articles). Finally, the GREMA group (see 3.7 below) of the IREM Paris sends its letter which has become the journal of the newly created International CII (cf <https://irem.univ-paris-diderot.fr/la-lettre-de-grema>).

In addition, many teaching manuals take advantage, with or without adaptation, of the activities developed in the IREM, and the production of the network appears in partner journals: *MathemaTICE*, edited par *Sésamath* (<http://revue.sesamath.net/>); *Recherches en Didactique des Mathématiques* (<http://rdm.penseesauvage.com/>), by the ARDM ; *Le Bulletin Vert* (<http://www.apmep.fr/-Le-Bulletin-Vert>) and *Au Fil des Maths* by the APMEP (<http://www.apmep.fr/-PLOT>); *Educmath*, by the IFE (French Institute for Education) <http://educmath.ens-lyon.fr/Educmath>.

3.3 Impact on didactics, epistemology and history researches

The research in mathematics didactics has emerged in France in the IREM, which have profoundly influenced its development. The work that Guy Brousseau has carried out for several decades at COREM, created by the IREM of Bordeaux at his initiative, is a particularly symbolic illustration of this (see interview <http://www.cfem.asso.fr/cfem/ICME-13-didactique-francaise>). These works have nurtured the development of the Theory of Didactic Situations. Similarly, those of Régine Douady, at the IREM of Paris, have developed the Dialectic of the Tool-Object and the Frame Games, those of Yves Chevallard at the IREM of Marseille, those of the Theory of Didactic Transposition, those of Raymond Duval at the IREM of Strasbourg, his Semiotic Theory, and those of Michèle Artigue at the IREM of Paris, Jean-Baptiste Lagrange at the IREM of Rennes and Luc Trouche at the IREM of Montpellier, the Instrumental Approach to Technological Integration. Régine Douady was president of the ADIREM and Michèle Artigue president of its Scientific Committee.

The IREM have had a profound influence on the didactic research conducted in France through their operating methods and values. They allow the researchers in mathematics education to stay in close contact with the mathematical community and nurture the recognized epistemological sensitivity of their research. They provide them with an effective contact with the field of teaching, that of the class, which reflects in their problems, theoretical constructions, as well as in the methodological importance given to the didactic engineering. Even when the institutionalization of didactic research led to the creation of specific research laboratories, in some universities, outside the IREM structure, these laboratories have kept close links with the IREM. And when the universitarization of pre-service teacher training (see 3.4) led many didacticians to migrate to the new training institutes, the links with the IREM remained strong. This is attested by the durability of the commissions COPIRELEM and CORFEM (see 1.2) both dedicated to the pre-service training. More generally, the National Didactics Seminar, the CII Didactics and the organization of researchers in didactics around the ARDM (see 1.3) have always worked in a very similar way. This impact on research is extended by other institutions (French Institut for Education - IFE) whose members can also work in the IREM.

The IREM have also promoted a collaborative and non-hierarchical vision of work between teachers and researchers. Their existence has allowed a percolation of research work among teachers that probably does not have an equivalent in any other discipline in France. The involvement of the IREM in in-service training is obviously essential. As is the work of applied research and of transposition of more

fundamental research findings, carried out within IREM groups and CII, and irrigating the training sessions they offer, and consequently the pre-service training of mathematics teachers (see Refforminit).

Since their creation, the IREM have also taken into account the teachers need to acquire a mathematics historical culture, which they often lacked, and to be provided with aids and resources to benefit from this culture. Specific working groups on these issues have thus been set up in many IREM. The mathematician and historian Jean-Luc Verley (IREM of Paris), from the beginning, wanted to propose to teachers and students the study of original mathematical texts; Jean Dhombres (IREM of Nantes, once director of this IREM, and president of the CS of IREM), was concerned to support multidisciplinary approaches and an epistemological reflection through historical work.

Thus, a specific research developed within the IREM, to support the introduction of a historical perspective into the teaching of mathematics from schools to university. From 1975, under the responsibility of Evelyne Barbin (IREM of Nantes) and Jean-Louis Ovaert, the CII "Epistemology and History of Mathematics" coordinated this work, quickly and durably becoming one of the most important commissions of the network. It still is today. It strives to disseminate the historical and epistemological works of the IREM to the mathematics teachers, but also to physical sciences and philosophy teachers, as well as to an enlarged audience of students and mathematics enthusiasts, through the conferences organized every two years on a specific theme (see <http://www.univ-irem.fr/spip.php?rubrique158>). The summer universities initiated in 1984 by the commission (<http://www.univ-irem.fr/spip.php?rubrique159>) became European summer universities, that alternate with the conferences of the History and Pedagogy of Mathematics (HPM) group affiliated with ICMI. Evelyne Barbin chaired the HPM group from 2008 to 2012. Thomas Hausberger and Anne Cortella organized the HPM 2016 conference at the IREM in Montpellier.

The work of the CII Epistemology and History, and more broadly of the IREM in this field, has resulted in numerous local and national publications: monographs on thematic topics of interest in mathematics education or grouping together exponents of the integration of their history into teaching, collections of ancient texts commentaries... In total, the CII itself has published some thirty books, edited by the IREM, the Institut National de Recherche Pédagogique (now IFE), university presses or private publishers. These works are internationally recognized, thanks in particular to the active participation of the members of the CII in European and HPM universities and symposia. The book *Passerelles : enseigner les mathématiques par leur histoire en Cycle 3* (teaching mathematics by their history, <http://www.univ-irem.fr/spip.php?rubrique505>) received the price of the sciences teaching book in 2019 of the French Sciences Academy. The CII also edited the book *Let history into the classroom* presented below 4.1

3.4 Impact on pre-service training

In-service teacher training has been the primary mission of the IREM, but they have also been involved in the pre-service training of future primary or secondary school teachers. In 1990, when this training was restructured in autonomous university institutes (IUFM), the IREM even were offered to become their research laboratories. Although they wished to remain inside the universities, as close as possible to the mathematicians, in order to maintain their founding principle of close interaction with mathematics research. Despite this, the IUFM mainly recruited their mathematics trainers among the IREM facilitators. Thus they were able to influence the pre-service training of teachers, some of which came in turn, once professional, to work in an IREM. This symbiosis has been facilitated when, in 2006, the IUFM were integrated into a university, then in 2013, were replaced by the ESPE, and finally in 2019 became INSPE (Superior Schools, then Superior National Institute of professorate and Education), with a growing weight of research in initial training. A great attention is paid to the durability of IREM's involvement in the new organisation of the studies of future teachers; ADIREM is working on this with the National Network of these structures.

Research, analysis of inventory of conditions, experimentation and critical reflection on the training of first and second degree teacher pre-service trainers are carried out in the IREM network by the commissions COPIRELEM, created in 1975 for the first degree, and the CORFEM created in 1993 for the second degree. Each of these CII organizes an annual "trainers' training" congress, hosted by an IREM (see <http://www.univ-irem.fr/spip.php?rubrique442> and <http://www.univ-irem.fr/spip.php?rubrique120>). These congresses are currently part of the National Teacher Training Plan.

An essential role of the IREM for future teachers is to convince them of the need to deepen their culture and professional practice throughout their careers; they facilitate their access to documentary resources (websites, libraries, etc.), and introduce them to use these tools and more generally offer them a framework adapted to the alliance of research and professional activity.

3.5 Impact on curricula

Since their creation, the IREM have been associated with the curricular reforms that have been successively carried out. This association has taken various and complementary forms, including:

- the driving role of the network in the creation and animation, from the 80's, of the successive national reflection commissions for long term thinking of mathematics education evolution. Best known is the CREM (<http://smf4.emath.fr/en/Enseignement/CommissionKahane/>, or Kahane Commission, from it's president Jean-Pierre Kahane, Academicien, past president of ICMI, past president of the Scientific Committee of the IREM;
- the regular participation of IREM facilitators, from secondary and higher education, in expert groups in charge of drafting mathematics curricula, or even the management of these groups, and the opinions systematically transmitted by the network on developed projects (cf 1.3);
- the privileged interaction of the National Education administration with certain CII: the CII highschool and vocational high school within an interdisciplinary scientific group for the ongoing reform; the COPIRELEM for the current implementation of university training for future teachers prior to their professional training(<http://www.univ-irem.fr/spip.php?article1456>);
- the experimentation of the curricula projects entrusted to the IREM: the series of the *Suivis Scientifiques* were accessible to teachers as soon as the college programmes were set up in 1985;
- the systematic implementation of training and the resource documents production to support curriculum development or changes in the functioning of the school system: by the CII Probability for high school statistics in 2000 and probabilities in 2009; recently with IREM's productions reworked along with the Inspectorate (see 1.3); by the multidisciplinary groups of the IREM for the gradual introduction since 2009 of interdisciplinary work in secondary school; currently, by the IREM for the creation of high school "mathematics laboratories";
- vigilant attention to the evaluation of the curricula consequences and of the educational environment, by supporting EVAPM (<https://www.apmep.fr/-Observatoire-EVAPM->, evaluation observatory of APMEP)

The pioneering work of the IREM has often directly influenced curriculum reforms: in 1981, the profound evolution of analysis teaching was supported by the work of the CII Analyse; for the current introduction of historical perspectives in math teaching by the CII Epistemology and History; for technological integration with the participation in the development of many math software programmes (Cabri-géomètre, Géoplan and Géospace, then Dgpad, or Xcas). Many experimental and research works are carried out in the IREM on numerical technologies, coordinated nationally by the CII TICE.

Since 2009, IREM has been supporting the introduction of algorithmics in mathematics programmes: in high school, then in elementary school and junior high school (2016). Many groups have been created on this theme, working with technology teachers, that have offered training (including University Diplomas: at Marseille and Grenoble). This led to create an algorithmics group in the CII high school. Then in 2017

the creation of a CII Informatique (or CIII), anticipating the introduction in 2019 of new informatics courses in high school, and the announcement for 2020 of a new status of informatics teacher.

3.6 Actions to popularize mathematics

In addition to their main activity of teacher training, the IREM have gradually developed popularization activities, towards both students and the general public. Towards the students, the well-known involvement of the IREM is through the regional mathematics rallies (19 in 2018), involving several hundred classes and several ten of thousands students. Some even exceeded borders: the Transalpin Rally, carried by the IREM of Franche-Comté, involving more than 4000 classes in 2018, from level 3 to 10, from Italy, French-speaking Switzerland, Belgium and Luxembourg. Teachers' meetings during rallies are also an opportunity to develop themes for in-service training. The problems proposed during these rallies and their analysis led to the publication of the "Panoramath" series, coordinated by the CII Pop'Math (7th volume in 2019, <http://www.univ-irem.fr/spip.php?article1555>).

The IREM cooperate with teachers and associations in actions to give students another vision of mathematics, encouraging scientific studies among girls and in modest social environments: *Math Kangaroo* (<http://www.mathkang.org/default.html>), the International Mathematics Games Committee's games fair (CIJM, <https://www.cijm.org/>), MATH.en.JEANS workshops and congresses or GirlsMath Actions or days (see 1.3). The IREM offer Hippocampus or MathC2+ internships (<http://www.irem.univ-mrs.fr/Hippocampe> and <http://eduscol.education.fr/pid23341-cid54958/mathc2.html>): reception of students inside university mathematics instituts, for several consecutive days to initiate them to research, by researchers, with or outside the classroom; students work on problems, experiment, debate and present their work as researchers. Initiated in biology by INSERM, the Hippocampus internships have been adapted to mathematics since 2005 by the IREM of Aix-Marseilles (15 internships per year) and now in Brest, Lyon, Toulouse...

The IREM participate in annual national actions for the dissemination of mathematics and its applications: "Sciences fair" (scientific activities fortnight for the general public and scholars), "Math Week" (under the auspices of the Ministry of National Education-MEN). In 2015, "Year of Mathematics for the Planet Earth", the IREM produced resources around this theme (<http://www.univ-irem.fr/spip.php?rubrique290>) and "Living Mathematics" forums were organized in Paris, Lyon and Marseilles, relying on the three IREM, coordinated by Michèle Artigue, then president of the IREM-CS, and Cédric Villani (<http://www.cfem.asso.fr/actualites/forum-mathematiques-vivantes>). Three other forums were held in 2017 in Lille, Rennes and Lyon. These forums mobilized several hundred teachers and researchers and welcomed many visitors. The network participates in 2019-2020 in the steering committee for the mathematics year, initiated by the CNRS for its 80th anniversary, organized jointly with the MEN (<http://www.cnrs.fr/fr/cnrsinfo/annee-des-mathematiques-rapprocher-chercheurs-et-enseignants>). The highlight will be a 3-days symposium in Lyon with 3000 teachers. Five exhibitions have also been created in the network, attracting teachers with their students and the general public: the exhibition *Regards sur les mathématiques- Itinéraires méditerranéens* (<http://www.irem.univ-mrs.fr/expo2013/english.html>, IREM of Aix-Marseille, english version created for HPM2016); the IREM of Grenoble is in charge of the animations and exhibitions of the museum *La grange des maths* (<https://www.echosciences-grenoble.fr/articles/la-grange-des-maths>); The IREM of Montpellier is broadcasting the exhibition "why mathematics?" created under the auspices of UNESCO with the participation of ICMI.

Finally, the network is strongly associated with other actors in scientific mediation and the promotion. The consortium *Animath'* (<https://www.animath.fr/>) participated in the CII Pop'math, as the society *Plaisir Maths* (<https://www.plaisir-maths.fr/>). Several working groups have been in common since 2017 between IREM and the *CultureMath* site of the ENS Paris and General Inspectorate (<http://culturemath.ens.fr/>), for teachers culture. Some members of the editorial committee of *Image des Maths* (<http://images.math.cnrs.fr/>) (CNRS website for the general public) come from the network.

3.7 The international network

Since the 70', IREM have collaborated with abroad universities to develop teacher training in order to enhance structures based on their model: first in French-speaking Africa, with an IREM in Madagascar, then in Dakar (Senegal, <https://irempt.ucad.sn/>), that became IREMPT (Math., Physics and Technology), or in Niamey (Niger). Currently, IREM-type structures are still opening up: the IREM/UPC in Kinshasa (Democratic Republic of Congo) in 2014, with the support of the GREMA group of the IREM of Paris (Reflexion Groupe for MATH, Teaching in Africa <https://irem.univ-paris-diderot.fr/groupe-irem/>). Others were born in Latin America: IREM of Lima (<http://irem.pucp.edu.pe/>, Peru), linked with the IREM of Caen, which organized its 9th international symposium on mathematics education (2018); IREM-Brazil, first based in Uniban, was created in 2008 (see 5.3).

The network is developing to promote trainers and researchers exchanges and the creation of collective resources. The PReNum-AC project - Production of Digital Resources for Secondary Math Teaching in Central Africa (<http://prenumac.free.fr/>, 2012-2015) aimed for training teachers in the use of teaching technologies (ICT), online tools, and mathematics didactics. It involved trainees and trainers from Cameroon and the Republic of Congo, and produced many resources for the 12 grade. PReNum-AC micro (2017-2018) followed it for the expedition of micro-servers for access to students without Internet connection to online exercise databases, PReNum-AC resources and software for the class.

Two congresses were held to federate activities in IREM or similar structures. In 2016, the international conference "Training of mathematics teachers, here and elsewhere" was held in Strasbourg, with 80 participants from more than ten countries. It marked the creation of the IREM international network and led to an international mailing list and a teleseminar (<http://www.univ-irem.fr/spip.php?rubrique455>). GREMA's letter has become the international network's letter. Algeria has since been involved creating IREM. Another meeting was held in 2018, on the margins of the French-speaking Mathematical Space (EMF) congress (<http://www.univ-irem.fr/spip.php?article1478>). Current events, projects, collaborations and difficulties were presented for Ivory Coast, Senegal, Madagascar, Algeria, Democratic Republic of Congo and Brasil. In Hungary, an IREM group has been created. It mirrors another at the IREM of Paris Nord, supported by the Hungarian Academy of Sciences. Joint seminars allow the facilitators of the two groups to meet and develop collaborations.

The resources exchange, and even of trainers, between IREM and equivalent structures abroad encourages the popularization of mathematics in these countries: hippocampus internships (see 3.6) are developed in the IMPA-CNRS International Joint Unit in Rio of Janeiro and in the University of Roma (<http://www.lix.polytechnique.fr/Labo/Matteo.Acclavio/ProgettoHippocampe.html>).

3.8 Openness to other sciences

Although recognized at the national and international level, no structures comparable to the IREM have been created for other sciences in France. Instead, it was decided locally to entrust IREM with tasks, particularly on STEMs, involving a close relationship between universities and the school world (valorisation, professional development actions, high school/university link).

If the development of groups on the history and philosophy of mathematics, and therefore also of sciences, was natural, it is mainly because of changes in mathematics curricula and from links, that became necessary, with teachers from other sciences, that multidisciplinary scientific groups have been created in the IREM (see 3.5). They support "interdisciplinary work in personal research" (in high school, then in junior high school), as well as the introduction of algorithmics and then computer science in curricula. Following the example of the IREMPT in Dakar (3.7), some IREM have included sciences in their missions (e.g. with the creation of a multidisciplinary group for the initial training of science teachers in Montpellier), some became IRES (for Science Education) : Toulouse and Orléans in 2015.

In 2016, the Ministry of National Education provided the network with additional resources to set up

research groups in other sciences, in particular to overcome the difficulties of science education in high school resulting from the 2009-2011 programmes. There are now 68 multidisciplinary groups working in the network and 16 groups without mathematicians. Physicists and/or chemists are involved in 37 groups, biologists/ecologists in 13 and computer scientists in 31. Several Universities are considering asking their IREM to become IRES or IREM&S, a change requested by the Conference of Science Universities Deans. In Poitiers IREM became IREM&S in 2017. The former IREM of Corsica will be reborn as an IRES. Particular attention is paid to keeping the IREM soul in this important evolution.

3.9 Role of the people involved in the IREM network

The functioning of the IREM network is not centralized. The presidents of ADIREM and CS (see 1.2) have important functions of management, coordination, dialogue with the authorities of ministries, academies and universities, and of representation of the network. But the network draws its strengths from its accessibility and the multiplicity and diversity of people organizing its activities: teachers at all levels, academics, researchers, teacher trainers. Their duties take many forms: organization of research groups, CII, trainings, publishing duties, digital information coordination, conferences and seminars organization, IREM direction, membership of boards (of an IREM, of the CS), scientific facilitation, etc. We can relate to several hundred people, and therefore several thousand since the IREM creation.

Despite the difficulties of tasks, often undertaken on top of teaching or research, and despite decrease in public subsidies, reduction in the means given by universities, and in the possibilities offered by the rectorships (for the presence of teachers in groups and in-service training courses...), it remains easy to find volunteers to suggest work themes or set up trainings, so strong are the conviction of the IREM's necessity and that their actions are carried out with great freedom.

We have chosen to present here "key persons" representing this diversity from the IREM rather than using standard academic recognition criteria. The nine "facilitators" briefly presented below have very different statutes and roles, and have had in the past, and still have an undeniable influence in different sectors of interest and the network.

Michèle Artigue - Emeritus professor at University Paris-Diderot- Member of the IREM network since the beginning of her academic career in the 70', she has been involved in diverse IREM activities from primary school to university. They have shaped her vision of research and practice in mathematics education, and her scientific production. She was director of the IREM of Paris from 1985 to 1988 and from 1999 to 2005, in charge of regional and national IREM projects, mainly on technological issues, member of the Scientific Committee of the IREM since 2004, and is the past-president of the committee. She has also contributed to the IREM network international visibility and to its involvement in international collaborative activities, especially with Latin America and Africa.

Yves Ducel - Maître de Conférences at the University of Franche-Comté (Besançon) - Involved in the activities of the IREM of Franche-Comté since 1991, he was Director from 1992 to 2000. Editor of the journal *Repères-IREM* and leading the associated inter-IREM Commission since 2002, he also was Deputy-Director, then Director (from 2002 to 2012) of the Presses Universitaires de Franche-Comté, and author of many papers and books on mathematics teaching and mathematics history. In 2011, under the aegis of the General Inspectorate of Mathematics, he took part in the production of official resources for the teaching of probability and statistics for grade 11.

Katalin Gosztonyi - Assistant professor at the Eötvös Loránd University of Budapest - She prepared her PhD in mathematics education under joint supervision in France and in Hungary, on the comparison of mathematics educational reforms in the two countries at the time of the New Math period. She started to participate in the activities of the IREM network during that period: she contributed to in-service teacher-education sessions and to the History and Epistemology group creation at the IREM of Paris-Nord. While continuing to participate in the activities of this group from a distance,

she has also created a teacher-researcher working group in Hungary, mirror to the previous one. Her research activities are deeply influenced by the work with these two groups.

Maelle Jouran - Mathematics teacher in junior high school - Member of the IREM of Rouen and of the inter-IREM Commission junior high school since 2005, co-chair of this commission since 2013, she co-organization several national conferences and co-edited the CII's last publication *Agrandir et réduire dans tous les sens (Enlarging and reducing in all directions)*. Teacher educator in the Academy of Rouen since 2006, she proposes every year professional development sessions in Academic Programme of Formation for junior high school teachers, based on IREM activities and resources. Since 2018, she contributes to the use of IREM resources in pre-service teacher education.

Hombeline Languereau - Mathematics teacher at the University of Franche-Comté (Besançon) - Member of the IREM of Franche-Comté since 1993, Director from 2004 to 2007 and vice-President of ADIREM from 2005 to 2007, she has been a member of the inter-IREM Commission "Epistemology and history of mathematics" since 1993, and has contributed to several publications in this area. Member of the inter-IREM Commission *Publimath* since 2008, she is currently its co-chair. Director of collection at the Presses Universitaires de Franche-Comté from 2004 to 2014, she is currently in charge of the Didactic series, which contributes to the dissemination of IREM productions.

Christian Mercat - Full Professor of didactics at the University Lyon 1 in the S2HEP laboratory and pre-service and in-service trainer - He has been Director of the IREM of Lyon since 2011 and has involved it into several European projects : Inter2Geo on interactive geometry, Math-Bridge on an intelligent remedial learning platform, mcSquared on technology fostering creativity in math teaching, MetaMath and MathGeAr, an international comparative study of math teaching for the engineer in Russia, Georgia, Armenia, Finland, Germany and France, DrIVE-MATH for innovative math teaching for the engineer and Mobile Math Trails for Europe (MoMaTrE) with geolocalized trails on smartphones. He belongs to the pilot committee of the House of Mathematics and Computer Science (MMI) in Lyon. He participated in the redaction of the Villani-Torossian report and takes part in a few high school "math laboratories".

François Moussavou - Mathematics and physics teacher in vocational high school in Marseilles - He has been member of the IREM of Aix-Marseilles since 2004, of the IREM Scientific Committee since 2014, member of the inter-IREM Commission Vocational high school since 2004 and chair of this commission since 2014. He has contributed to various working groups, to the production of official pedagogical resources and to professional development activities for mathematics and physics teachers in vocational and junior high school since 2005. In 2014, under the aegis of the General Inspectorate of Mathematics, he took part in the production of an official resource on the connection between general and professional disciplines for the vocational Baccalauréat. He is a member of national examination boards for the recruitment of teachers for vocational education since 2015.

Catherine Taveau - Mathematics teacher at the ESPE of Aquitaine (Ecole Supérieure du Professorat et de l'Education) at University of Bordeaux - Teacher educator since 1994, co-author of many resources for primary and secondary school mathematics teachers, she chaired for six years (2001-2005 and 2011-2013) the COPIRELEM, commission devoted to primary mathematics education, and has been a member of the IREM Scientific Committee from 2004 to 2012. From 2013 to 2016, she was involved in a research project on the experimentation of the "TI Calculator Primary+", carried out in partnership between some IREM and the French Institute for Education (IFE).

Dominique Tournès - Full professor of mathematics and history of mathematics at Laboratory of Mathematics and Computer Sciences of the University of La Réunion - He is researcher in the history of numerical analysis, mathematical instruments and mathematics of engineers during the period 1750-1950. For more than 30 years, he has been a pre-service trainer for mathematics teachers. Involved in IREM activities since 1982, he has been the director of the IREM of La Réunion since its creation in 1999, and a member of the inter-IREM Commission "Epistemology and history of mathematics" since 2000. He has chaired this commission from 2014 to 2017, and has contributed to 13 publications of this

commission (see 4.1).

4 Presentation of three publications that reflect the work of the network

We have chosen publications representative to different types of works we usually publish in the network: books, journals, and booklets (see 3.2).

4.1 Let History into the mathematics classroom - book - Springer - 2018

This book, published in 2018 in the series "History of Mathematics Education", is authored by teachers and researchers working in France in the IREM and members of the Inter-IREM Commission "Epistemology and history of mathematics". The ten chapters are English translations of a selection of texts published in France in two books produced by this Commission respectively in 2009 and 2012. the book aims make the work developed in France in this area more accessible to an international audience, beyond the French-speaking space.

Here are the titles and authors of the 10 chapters:

1. Angles in Secondary School: Surveying and Navigation, by Jean-Paul Guichard
2. Dividing a Triangle in the Middle Ages: An Example from Latin Works on Practical Geometry. by Marc Moyon
3. A Square in a Triangle, by Patrick Guyot
4. Indian Calculation: The Rule of Three—Quite a Story ..., by Catherine Morice-Singh
5. The Arithmetic of Juan de Ortega: Equations Without Algebra, by Frédéric Métin
6. The Congruence Machine of the Carissan Brothers, by Martine Bühler
7. A Graphical Approach to Euler's Method, by Dominique Tournès
8. Calculating with Hyperbolas and Parabolas, by Dominique Tournès
9. When Leibniz Plays Dice, by Renaud Chorlay
10. The Probability of Causes According to Condorcet, by Gérard Hamon

We copy here the introduction of the book, written by Evelyne Barbin, the editor, member of the CII Epistemology and History of mathematics, from the IREM of Nantes.

Introduction to the Book *Let History into the Mathematics Classroom*

This collection brings together ten experiments in introducing an historical perspective to mathematics teaching for 11–18 year olds. They suggest not only having students read ancient texts, but also to construct, draw and manipulate. History illuminates the present way beyond the mathematics already in the syllabus, through mathematical investigations that are both mental and manual. The different chapters of the collection refer to ancient Greek, Indian, Chinese and Arabic mathematics as well as to contemporary mathematics. They meet well-known mathematicians like Gottfried Leibniz or Leonard Euler as well as less famous practitioners and engineers, placing them in their scientific and cultural contexts.

One of the main values of history is to show that the notions and concepts we teach were invented to solve problems. The different chapters of this collection all have as their starting point historic problems—mathematical or not. These are problems of exchanging and sharing, of dividing figures and volumes as well as engineers' problems, calculations, equations and congruence. The mathematical reasoning which accompanies these actions includes the use of drawings, folding, graphical constructions or the production of machines. We have divided the chapters of this collection into four sections covering similar topics.

Describing and Measuring Magnitude

The first chapter involves the notion of angles and their measurement, based on problems of surveying

and navigation using measuring instruments such as the graphometer, sixteenth century portolans, maps, wind roses and maritime maps. The treatises on practical geometry are very useful as we have no written evidence of the first Greek geometers' investigations in the sixth century BCE. But we do have in our possession the famous Elements of Euclid, where geometrical knowledge is organised by axioms. The following chapter hinges on the problems of areas which underlie the whole of Book I of Euclid and the succession of hypotheses that lead to Pythagoras' Theorem. The aim of this is to expand not only the acquisition of geometrical knowledge but also its practice and demonstration. Physically cutting out figures to create others is the basis of many of Euclid's demonstrations. But it is not included for its own sake because it is subsumed, according to the precepts of Greek geometrical rigour, by the construction of divisions of figures possibly with the help of straight lines and circles (drawn with a ruler and compasses). The part on the Arabic Middle Ages shows us that this concept inherited from Greek geometry continues. But at the same time, works on practical geometry propose and justify useful divisions, for example by the craftsman who makes mosaics. Among the medieval authors, if the problems of dividing triangles find their place in geometrical constructions, they are also useful in problems of inheritance.

The chapter on drawing a square in a triangle illustrates the use of history to present the same problem in a great number of different ways. This approach should play an important rôle in teaching because it shows the vital essence of taught knowledge. It is not just a question of knowing how to solve a problem, but also to know how it was resolved in order to deal with new situations. With this problem, we can compare geometric solutions with the algebraic solution of al-Khwarizmi, and the students can use different types of instruments in class.

Calculating and Combining Numbers

The first chapter leads us to twelfth century India where the mathematician Bhâskara explains that the rule of three is as omnipresent in calculations as the god Vishnu is in the world. He shows how the rule of three is set out and how to carry out the calculations to solve all sorts of problems. But the history of the rule of three started long before and it carries on today. The following chapter, dedicated to a Catalan sixteenth century arithmetic book shows that proportionality is enough to solve problems that result in first degree equations today. In fact, the so-called methods of false position eliminate the need for algebraic calculation. It is undoubtedly for this reason, and also in view of their effectiveness, that they appear in merchants' calculations as well as in teaching until the start of the twentieth century.

The brothers Carissan's machine, presented in 1920 to the Société d'Encouragement de l'Industrie Nationale (Society for the Encouragement of National Industry) calculates arithmetical congruencies. This machine is a kind of "theorem in action," as it uses the algorithm of factorisation invented by Fermat in 1643. Its study by 18-year-old science stream students is contained in the final chapter of this section dedicated to numbers.

Drawing and Approximating Curves

The text of Leonard Euler (1768) introduces the problem of determining "in the most accurate way" the complete integral of any differential equation. Ballistics and celestial mechanics are the issues from which this arises. In the first chapter of this section Euler's famous method inspires a graphical approach in order to give greater meaning to the notion of the differential equation in upper secondary school.

In the same spirit, the subsequent chapter shows how nineteenth century engineers substituted graphs of curves which allowed the desired results to be read directly rather than use an abacus for tedious calculations. Graphs have many uses: they allowed the Austrian engineer Julius Mandl to solve equations from the second to the fourth degree. This chapter shows that nomograms (hyperbolic or parabolic) are useful nowadays as enriching activities for upper secondary school students.

Estimating and Predicting Chances

Probability lends itself well to an historical approach, starting with the "problem of points" which is at the start of the famous correspondence between Blaise Pascal and Pierre de Fermat. The chapter on the

problem of “quinquenove” comes from a text of 1678 by Gottfried Leibniz. One of the main interests of this text is to set probabilities against statistics. Another interest it offers is it contains an error in classic reasoning that is useful to point out to students. Here we see how the history of mathematics is an anchor point for epistemological reflection on rigour and error in mathematical activity. The last chapter involves a text by the Marquis de Condorcet on the application of calculating probabilities “in games of chance, in the lottery and in the judgement of men” dating from 1805. Reading this text can allow final year students to go beyond the frame of basic probability and explore a situation where statistics can be useful in analysing a phenomenon involving risk in hindsight whereas the problems arise in advance. The epistemological interest of the text is doubled through the cultural interest with regard to the author and his era.

The Introduction of Historical Perspective in Teaching

We speak in this collection of historical perspective because here it is not a question of either teaching the history of mathematics or even to place moments in history in their context, but to integrate history into teaching. Lots of the experiments presented here suggest getting students to read old texts. But the introduction of an historic perspective does not necessarily require such reading. More generally, it outlines the importance of teachers using all of their reflections (both historic and epistemological) in their teaching. Therefore, the reader will not find here a ready-made formula or a single answer.

The different experiments recounted clearly illustrate the variety of resources that a mathematics teacher can find in the history of their subject at all levels of teaching. Indeed, when the authors indicate the circumstances in which these experiments took place, it is to outline their situation and to invite the readers to adapt them or transfer them to other places, other classes or other levels. Many of these examples can be envisaged in classes other than those in which they first took place. This is because the programmes of study and the students change, but also, at a deeper level, because the history of mathematics allows for the exploration and highlighting of enduring knowledge which is a part of the common base of mathematics teaching.

The Authors and the Works of the French IREM

The authors of this collection are teachers and researchers working in France in the Institutes for Research in Education in Mathematics (IREM). They are sharing their methods and reflections and for a few moments are taking readers on a long mathematical adventure, which places knowledge before the challenges posed by problems of all kinds, mathematical and otherwise.

The Institutes for Research into the Teaching of Mathematics (IREM) were created at the end of the 1960s. They are University Research Institutes, dependent on a University and linked to the Mathematics Department of that University, and there is usually one Institute per Academy, which is the local education authority of the State Education system in France. The members of the IREM are university or secondary school teachers.

The Inter-IREM Commission “Epistemology and history of mathematics” was set up in 1975. Half the members are high school teachers while the other half are secondary school teachers and academics. Although the majority are mathematics teachers, there are also philosophy and physical science teachers, as well as researchers into the history of science. They work in the various IREM, where they belong to a research group in an IREM and organise education and training courses. This Commission meets two or three times a year in Paris and organises a conference every 2 years in a provincial city. The topics of these conferences correspond to current interests or work in progress. The proceedings and other material from each conference are published. The Commission also publishes books and anthologies.

The French version of the ten chapters of this volume appeared in two books edited by Évelyne Barbin and published by Vuibert Adapt-Snes (Paris) and titled *De grands défis mathématiques d'Euclide à Condorcet* (2009) and *Les mathématiques éclairées par l'histoire. Des arpenteurs aux ingénieurs* (2012).

4.2 Special issue of Repères IREM (112) July 2018

Each year, the journal *Repères-IREM* publishes a special issue on a current interest theme in mathematics education. We have selected the special issue published in July 2018 focusing on the links between mathematics and other disciplines.

The full text of this issue will be available on line in July 2019 by the link

http://www.univ-irem.fr/spip.php?rubrique24&id_numero=112.

One can for the moment access via this link to the editorial, the *Publimath* reports of each of the articles of the issue, and to some of the full texts.

Below, we propose translations of the description of the theme provided in the call for contribution, issued one year and a half before publication (this is the norm for this journal), and the editorial written by Anne-Cécile Mathé, member of the editorial committee, including a description of the content of this issue. Note that, as is now systematic in these special issues, it ends by a section "Pour aller plus loin" (To go ahead) referencing all articles addressing the theme published in the journal since its creation. In this particular case, 106 articles are listed, all accessible in line via *Publimath* except for the last 8.

Description of the special issue theme in the call for contribution, December 2016

In 1960, the physicist Eugene Wigner published a famous article with the provocative title "The Unreasonable Effectiveness of Mathematics in the Natural Sciences". Like Wigner, this special issue aims to show that the interactions between mathematics and other disciplines can be multiple and surprising. First, history tells us that most mathematical concepts come from a variety of disciplines: Philosophy, physics, chemistry, biology, geography, agriculture, economics, commerce, architecture, etc, are fields that have largely inspired mathematicians of all periods and civilizations. On the other hand, other disciplines have often drawn on mathematics to solve a problem, describe or model a particular phenomenon. We can thus speak of symbiosis as the ways back and forth between mathematics and certain fields are admirable! Similarly, through their aesthetic character, mathematics have forged many links with artistic disciplines such as painting, sculpture, music and literature. These links are studied in many fields: history, epistemology, philosophy, neuroscience, etc.

These interactions are increasingly important in secondary education programmes (IPE in junior high school, MPS in grade 10, TPE in grade 11) but also in university (history of mathematics education in baccalauréat degree, epistemology and literature courses in mathematics master degrees) and in the ESPE (history of maths courses in initial or in-service teacher education). Are they necessary to give meaning, help in the understanding and memorization of concepts, arouse motivation, develop curiosity? Conversely, it is also questionable whether these interactions are sufficiently taught, both in mathematics and in other disciplines.

In a non-exhaustive way, here are some questions likely to interest all the actors in education (vocational education, technical education, general education) but also all levels (primary, secondary, tertiary), on which we expect your thoughts and written contributions.

Editorial by Anne-Cécile Mathé, IREM of Clermont-Ferrand

To make mathematics interact with other disciplines, to overcome disciplinary divisions in order to achieve a global understanding of the world... Interdisciplinarity is regularly highlighted in curricula from primary school to high school. This movement is not new and trans-, pluri- or inter-disciplinary projects are flourishing in schools. The vast majority of teachers who try these practices express a definite enthusiasm for themselves and their students. However, achieving interdisciplinarity in the classroom is not an easy task. Such a project requires the implementation of a real collaborative work, often costly in time and energy, which alone can allow the understanding of the other, of its specificities and of the way in which the goals and means of each of our disciplines are likely to be articulated and enriched.

In this special issue "Mathematics and other disciplines", we wanted to highlight this movement and propose complementary and varied examples of such approaches.

The EPI group (Interdisciplinary Practical Teachings) at the IREM of Grenoble first presents an interdisciplinary project on the study of the evolution of glaciers. The authors demonstrate their desire not to confine mathematics to be a simple "toolbox" to the services of questions asked by other disciplines. They show the possibilities of a link between the problems specific to each of the disciplines and the complementarity of the insights provided, in order to study a major common question.

Taking a rather different perspective, Nicolas Saby and Cyril Tédéo then turn their attention to how mathematics education participates - or could participate - in the construction of tomorrow's citizens. These authors flaw the idea that the teaching of mathematical rationality is in itself a tool for training future citizens. They regret that the question of the transfer of the mathematical method to social and citizen issues is not more addressed by education. They then invite us to turn to social mathematics, to rediscover this historically neglected field of mathematics and to explore the avenues it offers us for teaching, in terms of corpus and Marion Quesney-Troudet, Marc Troudet and David Chatelon finally narrate the story of the encounter between the concerns of sport teachers around orienteering race activities and the work of an IREM group in Grenoble on the use of practical geometry instruments in class. An interdisciplinary project for eighth grade classes, aimed at orientation race practices, orientation knowledge, mathematical knowledge related to space and geometry, data organization and management or functions, was born from this meeting. The story of the genesis of this project is a good illustration of the work of inter-understanding the objectives and concerns of each of the disciplines necessary to bring interdisciplinarity to life in the classroom. This experience could be compared with an article published by Monique Maze and Guy Chataing on the orientation path as a support for work on angles in junior high school in Issue 2 of Repères-IREM. It will undoubtedly be able to arouse interest, ideas and desires.

In this issue, you will therefore find various examples of dialogues between mathematics and other disciplines, for a rich and open teaching of mathematics! Have a good reading!

4.3 Enseigner les mathématiques au cycle 4 à partir des grandeurs : les Prix (Teaching mathematics from quantities in Cycle 4: Prices) - booklet - IREM of Poitiers - 2018

Published in 2018, this booklet of the IREM of Poitiers, is part of a series including now 12 volumes produced by the junior high school group of this IREM, and proposing teaching progressions based on quantities. Each publication focuses on a specific quantity, prices in this particular case. According to the new curriculum organization, Cycle 4 corresponds to grades 7, 8 and 9, the three last years of junior high school. Within the framework of the AMPERES and PERMES projects aiming at the development of learning trajectories and teaching progressions obeying the model of Study and Research Paths (SRP) of the Anthropological Theory of the Didactic (ATD).

Link to the full texts of the series (in filigrane for the last volumes) of the series:

http://irem2.univ-poitiers.fr/portail/index.php?option=com_content&view=article&id=180&Itemid=197

An access is given to the Emma Castelnuovo award board to the on-line bank of documents accompanying the booklet

We propose below the translation of the introductory section of this publication, and give information about its referencing.

Translation of the introduction of this publication

"This is why everything must have a fixed price, because it is the condition for there to be always the possibility of exchanges, and therefore of association. Money is a kind of standard that makes things

commensurable and puts them on an equal footing. " (Aristotle, *Ethics in Nicomachean*, Book V)

The analysis done for Cycle 4 (Teaching mathematics in grade 7, 8 and 9 from quantities: Why? How?, pp. 3-7) led us to continue the study of Prices in Cycle 4 for the following reasons:

- It is a quantity omnipresent in human lives.
- This is the place where a majority of the content of the domains Organization and data management, functions, and Numbers and calculations of the curriculum can be used to compare, share, calculate prices, price variations, to study prices as function of another magnitude, and to forecast prices:
- statistics to study large amounts of data and make comparisons;
- percentages, fractions, proportionality to divide prices;
- powers to calculate and compare investments or loans, to deal with situations involving large numbers;
- relative numbers to compare or calculate losses or profits, to calculate price changes;
- probabilities to evaluate the chances of winnings;
- literal calculation and equations in situations involving formulas and the search for unknown prices (e. g. finding an initial price following a price increase or reduction);
- linear and affine functions, quotients and products quantities in the study of the variation of prices as functions of another quantity (duration, distance, volume, area, energy...).
- Algorithm programming also has its place: currency converters, price comparators, calculation of monthly loan payments, and quizzes abound on the Web.

The interest of this quantity Price is important. Much of the practical arithmetic that has been built up over the centuries, but also of elementary algebra, comes from problems related to trade, wages, taxes, inheritance, corporations and their profits (see Part 6. History). These problems are still real today. Prices are part of the students' environment and are omnipresent in our daily lives (see part 5. Ecology):

- to exchange, buy or sell, we often have to compare prices, and also to calculate prices according to...
- to pay a salary, to find the monthly payments to be paid, the amounts of reduction or increase, taxes, to share goods or objects we have to multiply and divide prices, to take fractions of them, to use proportionality and relative numbers;
- when you get information on economic and social life, you have to read graphs, compare prices: variations in prices, currency or share prices, comparison of salaries in different professions, budgets...

Prices are also often linked to other quantities: price per kilogram, per quintal or per ton, per km, per acre, per hour, per day, per week, per month, per year... This makes it possible to process the contents of the Sizes and Measurements section of the programme. With these products or quotient quantities, we are then confronted with many problems relating to proportionality, for which it will be necessary to develop tools and methods to solve them. Prices fluctuate, vary over time, periods, places, countries, which frequently leads to the construction of graphs and the use of equations. Many games of chance are related to the money you can expect to earn, which leads to an introduction to the basics of probability calculation. So we see that there is no shortage of situations, and that they revolve around a few fundamental questions:

- how to compare prices?
- how to share prizes?
- how to calculate prices?
- how to calculate price variations?
- how to study the variation of a price according to another quantity?
- how to anticipate a price?

Informed by these analyses, our chapters on prices in Cycle 4 are organized at each level as a journey around some of these 6 major questions. To make it easier for everyone to choose, we suggest that you see what knowledge of the programme can be invested in it, at what level, and how (2. Mathematical organization).

For each question, for implementation in the classroom we propose, examples of situations in people's lives, past or present (4. Situation bank). The study of these world situations will be all the more interesting as it will lead students to ask themselves questions and seek to answer them, in a problem-solving approach, where the knowledge of the curriculum will appear as modelling and solving tools (see programme, p. 367, §4). The choice of these situations and their questioning is therefore important. To organize the study of Prices, at each level of schooling, around a few key questions such as How to compare? How to share? How to calculate? allows you both to choose the situations to study, to structure the path of study of the quantity, and to organize the mathematical knowledge encountered as answers to these questions (3. Didactic organization).

The programme emphasises that the knowledge and skills targeted must be worked on gradually and reinvested throughout the cycle (p. 367, §1): this is one of the strong points of our approach (Teaching mathematics in grades 7, 8 and 9 from quantities: Why? How?, p.7, p.23, p.60).

Referencing

- The booklets of this series are referenced in the *Publimath* database <http://publimath.univ-irem.fr/> (launch the search with IREM de Poitiers), and their publication announced in *Repères-IREM* (114) http://www.univ-irem.fr/exemple/reperes/articles/114_parutions.pdf.
- 5 volumes of this series were listed among the 14 references proposed by the Ministry of Education in the document entitled "Mathématiques et quotidien" (Mathematics and daily life), one of the official resources produced to accompany the implementation of the new 2016 curriculum for junior high school: http://cache.media.eduscol.education.fr/file/Ressources_transversales/99/8/RA16_C3_C4_MATH_math_et_quotidien_600998.pdf.
- This series of publication is also used as a reference by researchers. For instance, in (Artigue & Blomøj, 2013)¹, the authors use one example from this series to illustrate what the ATD has to offer for conceptualizing IBME through the idea of SRP.

A presentation of this booklet was recently posted on the National Portal for Mathematics of the Ministry of Education (2019/05/20): <https://eduscol.education.fr/maths/actualites/actualites/article/enseigner-les-mathematiques-au-cycle-4-a-partir-des-grandeurs-les-prix.html>. We have translated it below:

Enseigner les mathématiques au cycle 4 à partir des grandeurs : les prix

An IREM publication.

The booklet *Enseigner les mathématiques au cycle 4 à partir des grandeurs : les prix* (PDF, about 7,8 Mo) is one of the recent publications of the IREM of Poitiers, and is part of the series of publications of this IREM proposing coherent teaching based on quantities.

This series well reflects the work of the IREM, at the interface between academic research and the terrain of school and teacher education. The authors rely on the notion of Study and Research Path coming from the Anthropological Theory of the Didactic and design SRP that are experienced in classrooms, reworked and then presented in an operational way in the booklets, where they are linked to the history and epistemology of the domains at stake, the curricula, and complemented by numerous resources allowing multiple adaptations. This series began in 2009 with booklets for grade 6, each concerning a specific quantity. The middle school IREM group then extended its work to the other middle school grades. This project benefited from the support of the Inter-IREM Didactic Commission and the IFE (French Institute for Education). These quality booklets are well distributed and the different volumes are regularly reissued. The paper versions are supplemented by online versions including many additional resources. Except for the most recent ones, a descriptive notice in the database *Publimath* exists.

Here are all the titles of the series:

- 2018. *Teaching mathematics in Cycle 3 from quantities: Angles*
- 2018. *Teaching mathematics in Cycle 4 from quantities: Prices*
- 2017. *Algorithmics and programming at Cycle 4 from quantities.*
- 2016. *Teaching mathematics in Cycle 4 from quantities: Lengths.*
- 2015. *Teaching mathematics in grade 7 from quantities: TEMPERATURES.*
- 2015. *Teaching mathematics in grade 7 from quantities: ANGLES.*
- 2014. *Teaching mathematics in grades 7, 8 and 9 from quantities. Why? How?*
- 2012. *Teaching mathematics in grade 6 from quantities: LENGTHS.*
- 2011. *Teaching mathematics in grade 6 from quantities: PRICES.*
- 2011. *Teaching mathematics in grade 6 from quantities: VOLUME.*
- 2010. *Teaching mathematics in grade 6 from quantities: DURATIONS.*
- 2010. *Teaching mathematics in grade 6 from quantities: AREAS.*
- 2009. *Teaching mathematics in grade 6 from quantities: ANGLES.*

5 Letters of support

5.1 Support of the French-speaking African Network for Research and Training in Didactics of Mathematics, to the IREM Network of France

The relationship between the IREM Network of France and those of Africa is 50 years old. They date back to the 1968-1969 academic year, which coincided with the creation of the first IREM in France. These relationships have been developed according to the respective institutional recommendations, taking into account their evolution over time. Our letter of support focuses on the principles and values that have underpinned this cooperation in the fields of education, pre-service and/or in-service teacher education, the production of resources for teaching or teacher education, research in the didactics of mathematics and mathematics education.

Support to overcome the shortage of professionals and resources

This support begins with African countries as soon as the first IREM are created in France. It takes place in a context marked by a pressing demand for education, a notorious shortage of professionals and a scarcity of documentary resources for those involved in teaching and teacher training in mathematics. The support system was characterised by two-way exchanges, through short stays on projects designed and implemented according to guidelines set by each of our countries. Thus, Senegal created the IREM of Dakar in 1972, which became an IREMPT in 1975 as an institute of the University of Dakar cooperating with the IREM of Montpellier, Lyon and Paris. Created in 1972, the IREM of Niamey has been organising the "Champagne Ardennes-Niger Rally" with the IREM of Reims since 2003. The Democratic Republic of Congo's collaboration with the IREM of Paris began in 1998; it was consolidated and renewed with the creation of the IREM in Kinshasa and the GREMA in 2014. The Brazzaville ENS, through the UREM, maintains a cooperation with the IREM of Paris-Diderot. Mali has opted for an association called the Groupe de Réflexion sur l'Enseignement des Mathématiques (GREM) created in 1975; in it, professionals in mathematics teaching, monitoring and evaluation and training collaborated in connection with the Ecole Normale Supérieure of Bamako; in 1991, an IREM-Grenoble/ENSUP-Bamako agreement was signed for the in-service and initial training of teachers in the didactics of mathematics.

Professional development in the field of mathematics education and teaching

The School of Didactics and Mathematics (EDiMaths) held in Bamako-Mali in September 2011, was the occasion for a fruitful partnership about the choice of training programme, of supervision or training system, and of the target audience. The second edition of this school took place in September 2012 at the Faculty of Science and Technology, Education and Training (FASTEF) - UCAD with the same support from the IREM network in France. The successes observed at the end of these two editions are the result of an approach based on a relevant combination of the expertise of facilitators working within the IREM Network of France, and that of trainers from the sub-Saharan sub-region, based on their experience in field practices. Thus, a sub-regional network has been set up to perpetuate the achievements of EDiMaths, strengthen connections and collaborations between the various communities involved in teacher education: mathematicians, didacticians, teacher educators at all levels of education, inspectors.

In the field of professional development, our links with the IREM network in France have enabled members of the network to register in the MOOC on the France Université Numérique (FUN) platform entitled "Enseigner et Former Avec le Numérique en Mathématiques" (EFAN-Maths). It offers distance learning for teachers, future teachers and teacher educators (see www.univ-irem.fr).

Mathematics education research - Increased visibility of African research

The IREM of France have played a major role in the implementation of inter-university cooperation agreements between Africa and French universities for doctoral studies and joint research projects in mathematics education. The missions to the IREM in France and vice-versa have been a source of motivation for several professionals from French-speaking African countries, for their definitive anchoring in the field of teaching, training and research in the didactics of mathematics. Similarly, the IREM International Seminar -Strasbourg-2016 and the IREM International Network Day, held in conjunction with the EMF-Paris-2018 Conference, provided a forum to disseminate the work carried out in Africa in this field. The implementation of the IREM tele-seminar, which made it possible to organize periodic teleconferences since 2016, has enabled five (5) of our fellow teacher-researchers to present their work and discuss it with other researchers. In addition, the IREM network in France has contributed to the establishment and functioning of African associations for scientific research in the field of mathematics didactics and mathematics education: the establishment of ADiMA and CANP1 are examples.

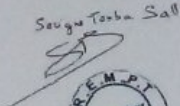
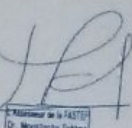
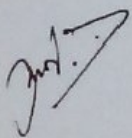
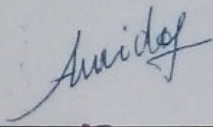
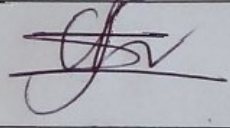
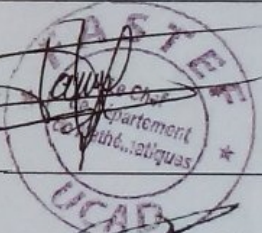
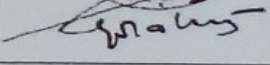
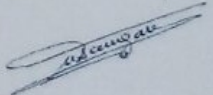
Support of the French-speaking African network to the IREM network in France

The network of mathematics education professionals in French-speaking Africa is particularly sensitive to the principles and values of the IREM network in France. This support is based, among other things, on the willingness to work together, the trainer, the teacher, the didactician and the mathematician being involved together and on equal terms, from primary school to university, in working groups where everyone works only part-time so as not to lose contact with the field and the associated expertise. This support is also based on the shared desire to develop in-service teacher education and to produce resources for teaching and teacher education rooted in this collective work, integrating research findings. It is being as close as possible to the daily needs of teachers and taking into account the diversity of contexts; with the desire to be a force for proposal for institutional decision-makers and also to respond to their legitimate demands, and with a spirit of collaboration and solidarity that transcends borders. It is around these values and principles that our respective professional careers are built to support mathematics education in our respective countries.

Consequently, the network of mathematics education professionals of French-speaking Africa hereby expresses its full support for the application of the French IREM Network for the Emma Castelnuovo Medal created by ICMI.

Signatories to this letter

Signataires de la présente lettre

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El Hadji Cheikh Mbacké DIOP- IREMPT – UCAD – Sénégal	
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Fernand MALONGA MOUNGABIO, Unité de Recherche sur l'Enseignement des Mathématiques Université Marien Ngouabi, Congo-Brazzaville	
Sidi Bekaye SOKONA, ENSup. DER Mathématiques, Bamako – Mali	
Mamadou Souleymane SANGARÉ ENSup. DER Mathématiques, Bamako – Mali	

5.2 Support of Gert Schubring (Brazil & Germany)



Universidade Federal do Rio de Janeiro
Centro de Ciências Matemáticas e da Natureza
INSTITUTO DE MATEMÁTICA

To
Konrad Krainer
Chair of the ICMI Emma Castelnuovo Award Committee

Rio de Janeiro, 24 May 2019

Re: Support to the candidature of the IREM for the Emma Castelnuovo-Award

Dear colleague,

I was very glad to learn that the French IREM are a candidate for the Emma-Castelnuovo-Award and I am supporting it wholeheartedly.

As a matter of fact, the net of the French IREM satisfy in a perfect manner the conditions for this Award: this net realises optimal achievements for the practice of mathematics education. From the creation of the first IREM in 1968, as an answer to the demands of the student movement to reorient science to the practice, to abandon the one-sided theoretical approach, the IREM succeeded in establishing an intense and always renewed strong cooperation between researchers in mathematics education and practitioners of mathematics teaching, the mathematics teachers.

There is not only this extensive cooperation with mathematics teachers within the structures of the IREM, by the groups "recherche-action"; there is, moreover, their strong participation in the mathematics teacher training. And besides this direct impact upon the practice of mathematics teachers, there is the broad and effective dissemination of the results of this cooperation to the large community of mathematics teachers in France, for the various levels of the school system. This dissemination is effected, in particular, by numerous and regular congresses, by differentiated types of journals and by easy access to the work of the IREM-net by the data base *Publimath*. The broad impact is also evidenced by the influence of the IREM on the elaboration of the mathematics syllabi for the school levels.

This long-standing and efficient practice of broad range achievements for the practice of mathematics teaching in France already clearly merits the Emma-Castelnuovo-Award.

I should like to emphasise, additionally, a further aspect evidencing the innovative character of this institutional net. One of the specific patterns of the IREM net is the work of Inter-IREM committees, for joint work on particular thematic issues of mathematics education. And one of them is the *Commission Inter-IREM Histoire et Épistémologie des Mathématiques*, working since the late 1970s. It is due to the continuous dedication of this Committee that the conception of using history of mathematics not only for deepening the mathematical formation of the teachers but also and foremost to introduce historical elements into the teaching of mathematics became a major innovation for the teaching practice. And it became

a key conception for this new approach to use original sources, giving thus access to the spirit and practice of former mathematics.

These works were always realised in close cooperation between researchers and mathematics teachers. An impressively great number of teaching materials and books were published over this period. Bi-annual conferences unite researchers and teachers – their title “*école d’été*” expresses very nicely the goal to learn there for later applying - and disseminate the results for use in classroom. The title of the volume with a collection of French contributions to this use of history, *Let history into the mathematics classroom*, published by Springer in 2018, expresses this conception neatly. As I was learning recently, history of mathematics will become a topic in the new mathematics syllabi. Likewise significantly, the *Académie des Sciences* has distinguished the book *Passerelles: enseigner les mathématiques par leur histoire au cycle 3*, elaborated by the Committee as a source for the mathematics teachers to practice the use of history in their classrooms, by the Award “*prix du livre de l’enseignement scientifique*” for 2019.

Introducing history of mathematics into mathematics teaching turned out not only to change the culture of the mathematics teachers and the classroom practice in France, it effected an enormous impact upon the international developments of mathematics education. Converging with a movement in England for the use of history in the teaching of mathematics, the French Inter-IREM group contributed decisively to establish a broad international cooperative work for this innovative strand. With it, the HPM emerged – The International Study Group on the relations between the HISTORY and PEDAGOGY of MATHEMATICS -, supporting this movement by regular newsletters and conferences. The volume *History in Mathematics Education. The ICMI Study* (eds. John Fauvel & Jan van Maanen; 2000) , result of an ICMI Study prepared by a conference at the CIRM in 1998, became the standard reference work for the use of history for teaching.

And one more important initiative of this French Inter-IREM Committee has to be evoked, being an enormously successful international activity for practicing the use of mathematics: it was in 1993, at Montpellier, that took place the first *Université d’Été Européenne d’Histoire et Épistémologie des Mathématiques dans l’Éducation Mathématique*. This turned out to become a very important series of conferences, occurring all three or four years, with an enormous number of mathematics teachers participating. Having addressed from its beginnings participants from Europe, it became eventually renamed in English as ESU, or *European Summer University History and Epistemology in Mathematics Education*, but still with the IREM Committee as its supporting structure. Last year, in 2018, took it place as the – already – ninth ESU.

Resuming, the French IREM deserve perfectly the Emma-Castelnuovo Award and I am strongly recommending to distinguish them with this Award.

With kind regards



(Universität Bielefeld/Universidade Federal do Rio de Janeiro)

5.3 Support from the IREM of Brazil



To

Konrad Krainer

Chair of the Emma Castelnuovo ICMI Award

From

Tânia Maria Mendonça Campos

Emeritus president of the IREM-Brasil **Marlene Alves Dias**

Coordenadora do IREM-Anhanguera

Universidade Anhanguera

Verônica Gitirana, Valdir dos Santos Júnior, Cristiane Rocha, Rogério Ignácio,

Iranete Lima

Grupo Pro-IREM-UFPE

Universidade Federal de Pernambuco

Wagner Rodrigues Valente

Coordenador do GHEMAT-IREM-UNIFESP

Universidade Federal de São Paulo, Campus Guarulhos, SP

Letter of support for the application of the IREM International Network to the Emma Castelnuovo Medal.

Nowadays, Brazil has three IREM groups - IREM-Anhanguera, GHEMAT-IREM-UNIFESP, and Pro-IREM-UFPE - which together constitute IREM-Brazil. Representing IREM-Brazil, we are sending this letter to lend our unreserved support to the IREM International network application for the Emma Castelnuovo Medal.

Fifty years-old, the IREM Network covers French territory, and it is nowadays making a great effort towards building an international network. Today associated structures exist in nine other countries and work in line with the IREM spirit and values, bridging academic research in mathematics education and history, and schools, considering in equal respect teachers, teacher educators and researchers who work there in a collaborative way, and developing a diversity of projects and publications that propagate mathematics teaching research and experiences.

IREM-Brazil is one of these structures. It started in 2008, in a jointly initiative of Prof. Michèle Artigue and Prof. Tânia Maria Mendonça Campos, following a collaboration with the IREM of Paris started in the late eighties. Based in Uniban, currently IREM Anhanguera, according to its own status and in the framework of the IREM missions, IREM-Brasil has developed its actions in four essential directions:

- Research on mathematics teaching and on research training for teachers from all school levels;
- Preservice and in-service mathematics teacher education;

- Production, experimentation, publication and diffusion of documents, software programs, pedagogic products, for teaching and teacher training;
- Contribution to the IREM International Network work.

In 2018, two other research groups of Brazilian universities having also links with French universities and IREMs, especially through CAPES-COFECUB cooperation projects and the High Studies Schools undertaken in diverse universities, such as UFPE expressed the desire to create similar structures and to join IREM-Brazil. The GHEMAT-IREM-UNIFESP was created and the other group Pro-IREM-UFPE is still in the process of being constituted as IREM.

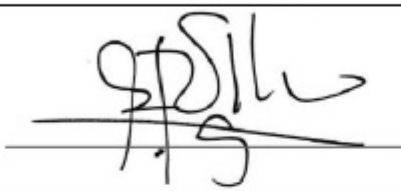
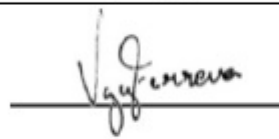
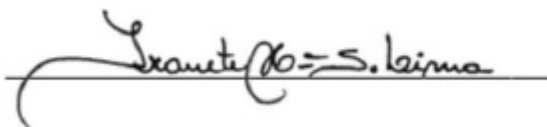
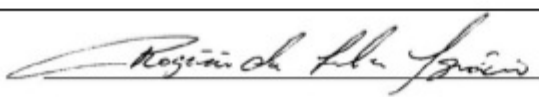
During all this time, many initiatives of IREM-Brazil, and from the groups that nowadays constitute IREM-Brazil, end-up with trainings for research of masters and doctors, many of them acting on different schools levels, particularly, on university level. It contributes with pre-service and in-service teacher education through cooperation between the universities and State Education Bureau in São Paulo and Pernambuco states, through different projects, such as Observatory of Education funded by the Ministry of Education. The results of research and training are disseminated through seminars, journals and meetings in the State Education Bureaus, in regional offices located in different places of the states or in the university itself. The national and international publications have contributed to disseminate the outcomes of the research and training carried out in Brazil, as well as in IREM-Brazil. In all these activities and publications, the influence of French didactic theories and of innovations developed within the scope of the international IREM network is clear.

The work that has been carried out by these different Brazilian groups improved the contacts and internal exchanges. This opportunity to work together through network actions, considering a rich set of training activities at the various levels of schooling as already being promoted in IREM-Brazil entities - those already constituted and those in the process of formalization - aggregates national and international researchers, teachers of all levels, students and post-graduate teachers. It contributes to the training of a substantial number of master and doctorate students about current themes associated with the teaching of mathematics. IREM actions have also helped in the changes that have been proposed so that the Brazilian students present better results in the national and international evaluations, since the international evaluations could be analyzed through different cultural perspectives.

Due to the results, we have obtained through the reflections carried out jointly with the support of the researchers related to IREMs and the documents and materials that we can adapt to our reality, we consider especially important the role of the IREM International network for the development of an effective teaching of mathematics that goes beyond the borders of French schools and universities.

Based on this long-term internationally cooperation and influential achievements, the IREM-Brazil highly supports the nomination of IREM International Network for the 2019 Emma Castelnuovo Medal.

Brazil, May 28, 2019

<u>Tânia Maria Campos</u>	Tânia Maria de Mendonça Campos Emetitus President of IREM-Brazil
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6 Coordinators of the case

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